Revenue-Constrained Combination of an Optimal Tariff and Duty Drawback

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December 2015

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Abstract
A duty drawback is an export subsidy determined as a percentage of the tariffs paid on the imported inputs used in its production. This paper examines the revenue-constrained optimal tariff structure in a small open economy including a duty drawback as a trade policy tool. This paper has two main aims. First, we show that the revenue-constrained optimal combination of tariff and duty drawback for a given revenue level is not unique. Second, we show that if the optimal import tariff rates are all positive when the duty drawback rate is zero, then the optimal import tariff rates are always positive when the duty drawback is positive.

Keywords: Revenue-Constrained Optimal Tariff, Duty Drawback, Export Subsidy, Uniformity

JEL classification: F11, F13, H21

1. Introduction
Many countries continue to use import tariffs as a major instrument for collecting

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1 This paper is a by-product of earlier work in Hatta and Ogawa (2007), for which the author gratefully acknowledges the detailed comments generously provided by Professor Ogawa. The usual disclaimer applies.
revenue as well as for protecting domestic industries.\(^2\) This is especially because some developing countries find it difficult to manage broad-based taxes, such as commodity, value-added, or income taxes, because their administrative and political costs are relatively high. In contrast, tariffs can be easily imposed and as a result are the main source of revenue in several countries, as shown by Greenaway and Morrissey (1996) and Ebrill, Keen, Bodin and Summers (2001).

A duty drawback is an export subsidy determined as a percentage of the tariffs paid on the imported inputs used in its production. As with an export subsidy, a duty drawback will stimulate exports by offsetting the export-restraining effects of tariffs on the imported inputs. Indeed, if provided, a full duty drawback would allow export industries to obtain imported inputs at world prices. Thus, the World Bank permits export duty drawbacks as an allowable trade policy (see Krueger and Rajapatirana (1999)), with the result that they now prevail in many developing countries.\(^3\)

Nevertheless, just a few studies to date have analyzed the effects of duty drawbacks. Of these, Panagariya (1992) examined the welfare effects of revenue-constrained tariff reforms under duty drawbacks and characterized some welfare-improving tariff reform programs, which are particularly useful for actual tariff reform in developing countries.\(^4\) More recently, Ianchovichina (2007) analyzed the welfare effects of tariff and duty drawback reform by extending the analytical framework in Panagariya (1992). However, for the most part, this literature has not addressed the goal of revenue-constrained tariff reforms under a duty drawback, i.e., the structure of reference revenue-constrained optimal tariffs under duty drawbacks has not been analyzed. Moreover, this literature has not considered the key question of precisely how duty drawbacks affect the revenue-constrained optimal import tariff structure.

In this paper, we examine the optimal combination of import tariffs and duty drawbacks on the export good under a revenue constraint. This serves as an extension of

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\(^2\) For example, the revenue obtained from import duties as a proportion of total tax revenue was 44.8% for Madagascar in 2000, 54.7% for Swaziland in 2000, 50.2% for the Bahamas in 2001, and 49.3% for Uganda in 2000 (IMF, 2002). Especially in many other African countries, it can exceed 30%.

\(^3\) Michalopoulos (1999) showed that many developing countries impose at least some form of duty drawback on exports.

\(^4\) Michael, Hatzipanayotou and Miller (1991), Neary (1998), and Keen and Ligthart (2002) theoretically analyzed tariff and tax reform as a revenue problem, but did not allow for duty drawbacks.
the theory of a “revenue-constrained optimal tariff” that has been well developed in the tradition of optimal commodity tax theory by Dasgupta and Stiglitz (1974), Heady and Mitra (1987), Mitra (1992, p. 246), Dahl, Devarajan and van Wijnbergen (1986), Panagariya (1994), Hatta (1994) and Hatta and Ogawa (2007). These studies characterize the tariff combination that maximizes utility when a fixed level of the revenue has to be collected only from tariffs\(^5\) in the situation where price distortions arising from the tariffs are inevitable.\(^6\) Thus, the revenue-constrained optimal tariff problem, which can arise even in a small country, entirely differs from the more familiar “optimal tariff problem in a large economy.”

In the present analysis, we introduce a duty drawback into the theory of the revenue-constrained optimal tariff.\(^7\) We consider a small open economy with three tradable goods (one import good, one export good, and one imported input), where a fixed level of revenue is collected from the combination of tariffs and the duty drawback.\(^8\) We show the following. First, an equilibrium supported by a combination of tariffs and a duty drawback can also be supported by infinitely many other combinations of tariffs and duty drawbacks yielding identical revenue. This implies that the revenue-constrained optimal combination of tariffs and duty drawback, we require at least one tariff or the duty drawback fixed at a given level. Second, we show that as the duty drawback increases, the associated optimal import tariff rates of all goods monotonically increase.

The remainder of the paper is organized as follows. Section 2 analyzes the structure of optimal import tariff rates and an export subsidy rate. This analysis is useful for examining the effect of a duty drawback as a conditioned export subsidy. Section 3

\(^5\) In an optimal commodity tax model, labor supply is endogenous and there is a distortion generated between goods and leisure. Commodity taxes and wage subsidies at a uniform rate would remove this distortion. However, tax revenue would then be zero. At this point, the optimal commodity tax problem for positive revenue becomes a concern. See, for example, Auerbach (1985) for a detailed description of optimal tax models. In the optimal tariff model, we assume a fixed labor supply, and hence can disregard this distortion.

\(^6\) If there is a subsidy for the export good at the same rate as the import tariff rate, there is no price distortion on tradable goods. However, under this tariff structure, all revenues collected from import tariffs are spent on the export subsidy, i.e., tariff revenue is zero. Consequently, to collect revenue from tariffs, price distortion is inevitable.

\(^7\) Cadot, de Melo and Olarreaga (2003) empirically examined the implications of a duty drawback in a political economy framework, but did not impose a revenue constraint. Ianchovichina (2007) and Chao, Yu and Yu (2006) showed via simulation analysis that duty drawbacks improve the level of welfare in China.

\(^8\) Note that a duty drawback yields negative revenue.
presents the model employing a duty drawback as trade policy and studies the structure of optimal combination of import tariff rates and the duty drawback. Section 4 derives qualitative relationship between the duty drawback and the optimal import tariff rate. Finally, Section 5 presents the exact formula of the import tariff rates as functions of the duty drawback, without assuming the Leontief type technology with rigid proportions of inputs and outputs.

2. The general tariff economy
   a. Nonuniqueness of the equilibrium price vector

   In this section, we consider an economy with import tariffs and an export subsidy. This prepares us for the following section, where we introduce a duty drawback by extending the model in this section. More concretely, we review the model of the General Tariff Economy (hereafter, GTE) introduced by Hatta and Ogawa (2007). The GTE is a small open economy with a revenue constraint arising from tariffs and subsidy. This economy has three tradable goods: good 0 is an export good and goods 1 and 2 are import goods. All of our notations follow Hatta and Ogawa (2007), such that \( q' = (q_0, q_1, q_2) \) is the domestic price vector, \( p' = (p_0, p_1, p_2) \) is the world price vector, \( t' = (t_0, t_1, t_2) \) is the specific tariff vector, \( x' = (x_0, x_1, x_2) \) is the demand vector, \( y' = (y_0, y_1, y_2) \) is the supply vector, and \( r \) is the tariff revenue. In the GTE, the following holds:

\[
q = p + t, \quad (1)
\]

\[
x_i = x_i(q, u), \quad i = 0, 1, 2,
\]

\[
y_i = y_i(q), \quad i = 0, 1, 2,
\]

\[
z_i(q, u) = x_i(q, u) - y_i(q), \quad z'(q, u) = (z_0(\cdot), z_1(\cdot), z_2(\cdot)),
\]

\[
p'z(q, u) + r = 0, \quad (2)
\]

\[
q'z(q, u) = 0, \quad (3)
\]

where \( x_i(q, u) \) is the compensated demand function, \( y_i(q) \) is the supply function, and \( z_i(q, u) \) is the compensated excess demand function for the \( i \)-th good. Equations (2) and (3) indicate the international balance of payments and the private sector budget equation, respectively. As the GTE is a small open economy, the world price vector \( p \) is constant.

3
We assume \( p_0 = 1 \).

When Equations (1), (2), and (3) are all satisfied, we say that the GTE is in equilibrium. When the vector \( (q^*, x^*, y^*, u^*) \) satisfies (1), (2), and (3), we refer to it as an equilibrium vector of the GTE, to \( (x^*, y^*) \) as its equilibrium allocation, and to \( q^* \) as its equilibrium price vector. Further, if two equilibrium vectors share the same equilibrium allocation, we say that these equilibrium vectors are equivalent, and that their equilibrium price vectors are equivalent. Thus, if vectors \( (q^*, x^*, y^*, u^*) \) and \( (q^{**}, x^*, y^*, u^*) \) are both equilibrium vectors of the GTE, then they are equivalent equilibrium vectors, and \( q^* \) and \( q^{**} \) are equivalent equilibrium price vectors. We should note that the levels of welfare and tariff revenue are both constant under equivalent price vectors because the equilibrium allocations associated with them are identical.

Given \( x_i(q, u) \) and \( y_i(q) \) are homogeneous of degree zero with respect to \( q \), a proportional increase in \( q \) does not affect the values of \( x_i(q, u) \) and \( y_i(q) \), keeping the equilibrium allocation of the GTE intact. Therefore, when \( q^* \) is an equilibrium price vector of the GTE, the vector \( \kappa q^* \) is also an equilibrium price vector of the same model for any positive number \( \kappa \). This implies that

\[
\kappa q^* \text{ and } q^* \text{ are equivalent equilibrium price vectors of the GTE} \tag{4}
\]

b. Nonuniqueness of the equilibrium tariff vector

The ad valorem tariff rate \( \tau_i \) of the \( i \)-th good may be defined by

\[
\tau_i = \frac{t_i}{q_i}, \quad i = 0, 1, 2. \tag{5}
\]

Then the ad valorem tariff vector \((\tau_0, \tau_1, \tau_2)\) corresponding to the price vector \((q_0, q_1, q_2)\) can be expressed as

\[
(\tau_0, \tau_1, \tau_2) = \left(1 - \frac{1}{q_0}, 1 - \frac{p_1}{q_1}, 1 - \frac{p_2}{q_2}\right). \tag{6}
\]
When \((q^*, x^*, y^*, u^*)\) is an equilibrium vector of the GTE, so is \((\kappa q^*, x^*, y^*, u^*)\) from (4). Hence the tariff vectors

\[
\left(\tau^*_0, \tau^*_1, \tau^*_2\right) = \left(1 - \frac{1}{q^*_0}, 1 - \frac{p_1}{q^*_1}, 1 - \frac{p_2}{q^*_2}\right) \quad \text{and}
\]

\[
\left(\tau^*_0, \tau^*_1, \tau^*_2\right) = \left(\frac{1}{\kappa q^*_0}, \frac{1}{\kappa q^*_1}, \frac{1}{\kappa q^*_2}\right)
\]

(7)

are equivalent. Thus, the tariff imposition at any choice of \(\kappa\) in (7) does not affect the equilibrium allocation \((x^*, y^*)\). Therefore, when \(\tau^*_0\) is chosen, Equation (7) determines the vector \((\tau^*_0, \tau^*_1, \tau^*_2)\) equivalent to \((\tau^*_0, \tau^*_1, \tau^*_2)\), and we have the following:

**Proposition 1.** The tariff vector that supports a particular equilibrium allocation of the GTE for a given tariff revenue is not unique.

This proposition is an extension of the Lerner Symmetry Theorem established by Lerner (1936), which states that there is always an equivalent rate of export tariff for any import tariff rate. The Lerner Symmetry Theorem straightforwardly implies many possible combinations of import and export tariffs that yield the same revenue and support the same domestic price vector.

c. **Nonuniqueness of the revenue-constrained optimal tariff vector of the GTE**

In this section, we consider the optimal tariff rate of the GTE. We can say that the tariff structure \((\tau_0, \tau_1, \tau_2)\) is a revenue-constrained optimal tariff vector if this structure maximizes \(u\) in the GTE model for a given revenue level. It is obvious that Proposition 1 continues to hold even when the tariff vector is optimal. Thus, we have the following:

**Proposition 2.** The optimal tariff vector of the GTE for given revenue is not unique.

3. **The duty drawback economy**

a. **Nonuniqueness of the equilibrium price vector of the duty drawback economy**

This section introduces a duty drawback on the export good into the GTE by introducing some structure to the GTE model. Thus \(x_2(q, u) \equiv 0\) holds.
Assumption 1. Good 2 is an imported input used to produce both goods 0 and 1. It is not a consumption good and not produced at home.

The imported input enters the output as a negative element, that is, \( y_2 < 0 \) and hence \( z_2 = -y_2 > 0 \).

Assumption 2. A duty drawback is given to an exportable good as a percentage of the tariff paid on the imported inputs used in that export. No other tariffs or subsidies are given.

The duty drawback \( \beta \) is defined by

\[
\beta = \frac{t_0}{t_2 a},
\]

where \( a \) is the amount of the imported input used in one unit of the export, i.e.,

\[
a = \frac{y_2(0)}{y_0},
\]

where \( y_2(0) \) denotes the amount of the imported input used in the production of the export good.

Because the factor demand and supply functions depend only on the domestic prices of the goods in equilibrium, we express the imported input ratio function of the export good by

\[
a = a(q).
\]

If there is no duty drawback, then from (8) \( \beta = 0 \) and \( t_0 = 0 \). Conversely, if \( \beta = 1 \), the duty drawback is full and export industries can access imported inputs at the world price. Indeed, it follows from (8) that \( t_0 = t_2 a \). The relationship between the world price and the domestic price on the exportable is given by

\[
q_0 = t_0 + 1.
\]

When \( t_2 > 0 \) and \( \beta > 0 \), (8) implies that \( t_0 > 0 \). This shows that a positive duty drawback plays the same role as an export subsidy. The duty drawback, like an ordinary export subsidy, raises the price of the exportable good and thereby protects the export
The GTE including Assumptions 1 and 2 is regulated to as the duty drawback economy (DDE). The DDE model consists of Equations (1), (2), (3), (8), and (9). When the vector \((q^*, x^*, y^*, u^*)\) satisfies Equations (1), (2), and (3) for the DDE model, we say that the DDE is in equilibrium, and the vector \((q^*, x^*, y^*, u^*)\) is an equilibrium vector of the DDE. Equations (8) and (9) yield the corresponding level of the duty drawback in this DDE. Therefore, the DDE is simply a GTE with the definition of \(\beta\) added. When \((q^*, x^*, y^*, u^*)\) is an equilibrium vector of the DDE, \((x^*, y^*)\) is its equilibrium allocation and \(q^*\) is the equilibrium price vector of the DDE.

If two equilibrium vectors of a DDE share the same equilibrium allocation, we can say that these equilibrium vectors are equivalent and that their equilibrium price vectors are equivalent. Thus, if vectors \((q^*, x^*, y^*, u^*)\) and \((q^{**}, x^*, y^*, u^*)\) are both equilibrium vectors of the DDE, then they are equivalent equilibrium vectors, and \(q^*\) and \(q^{**}\) are equivalent equilibrium price vectors of the DDE. We should note that the levels of welfare and tariff revenue are both equal under the equivalent price vectors because the equilibrium allocations associated with them are identical.

When \((q^*, x^*, y^*, u^*)\) satisfies Equations (1), (2), (3), (8), and (9) for some \(\beta\), it is called an equilibrium vector of the DDE for the given \(\beta\). Note that \(a(q)\) in (9) is homogeneous of degree zero with respect to \(q\). Hence, Statement (4), established for the GTE, is valid even for the DDE given (4) was derived from (1), (2), and (3). Thus, we have

\[ \kappa q^* \text{ and } q^* \text{ are the equivalent equilibrium price vectors of the DDE} \]

b. Nonuniqueness of the equilibrium tariff–duty drawback vector of the DDE

We may rewrite Equation (8) in terms of the ad valorem tariff rates:

\[ \tau_0 = \alpha \beta \tau_2 \text{ and } \alpha = aq_2 / q_0, \]

where \(\alpha\) is the payment for the imported input used in one dollar of the exported good. Assumption 1, which states that the imported input produces both goods 0 and 1
implies that $0 < \alpha < 1$.

Given the variable $\alpha$ is homogeneous of degree zero with respect to the domestic prices, $\alpha$ is constant under a proportional change in domestic prices. From (5), (11), and Statement (10), we have the following:

**Proposition 4.** The duty drawback–tariff vector $(\beta, \tau_1, \tau_2)$ that supports a particular equilibrium of the DDE for a given revenue is not unique.

c. **Nonuniqueness of the revenue-constrained optimal duty drawback–tariff vector of the DDE**

We say that $(\beta, \tau_1, \tau_2)$ is the revenue-constrained optimal duty drawback–tariff vector of the DDE if this structure maximizes $u$ in the model of the DDE for a given revenue level. It is obvious that Proposition 4 continues to hold even when the tariff vector is optimal. Thus, we have the following:

**Proposition 5.** The optimal duty drawback–tariff vector $(\beta, \tau_1, \tau_2)$ of the DDE for a given revenue level is not unique.

This proposition implies that a policy prescription valid in an economy with a lump-sum tax does not necessarily apply in the DDE, where the total revenue from tariffs is fixed. For example, Proposition 3 in Panagariya (1992) showed that in a three-commodity model similar to the DDE model, raising the duty drawback rate from zero accompanied by an increase of the tariff rate on the imported input will necessarily improve the welfare of the economy, if the lump-sum tax is adjusted behind the scenes. However, this proposition does not necessarily hold in our fixed revenue economy. From Proposition 5, we can choose an optimal duty drawback–tariff vector in such a way that $\beta$ is zero. Thus suppose that the initial level of $\beta$ in the optimal duty drawback–tariff vector is zero given. Then an increase in $\beta$ from zero, while $\tau_2$ is adjusted to keep revenue constant and $\tau_1$ fixed, will necessarily reduce welfare if the starting duty drawback–tariff structure was optimal.

Another implication of the proposition is that the level of a duty drawback itself can be set at a politically or administratively convenient level as long as the import tariff rates are
optimally adjusted. In other words, it is possible to keep the tariffs on the imported input high as long as it is possible to raise the duty drawback politically. On the other hand, if a nonzero duty drawback is politically or administratively infeasible, then the import tariffs can be selected optimally to suit the zero level of a duty drawback.

4. Import tariff rates as a function of the duty drawback rate

a. Import tariff rates as a function of the export tariff rate in the GTE

From equation (7), we can express two import tariff rates as functions of an export tax rate so as to keep the equilibrium resource allocation intact.

Let \((q_0^*, q_1^*, q_2^*)\) be an equilibrium price vector and \((x^*, y^*)\) be the equilibrium allocation achieved at \((q_0^*, q_1^*, q_2^*)\). Then, the equilibrium \((x^*, y^*)\) can be attained by the tariff vector \((\tau_0^*, \tau_1^*, \tau_2^*)\) defined by (7) for any given \(\kappa\). For simplicity, we write the equilibrium tariff vector \((\tau_0^*, \tau_1^*, \tau_2^*)\) corresponding to \((x^*, y^*)\) as \((\tau_0, \tau_1, \tau_2)\) hereafter.

**Lemma 1.** Then, for any value \(\tau_0\), the equilibrium \((x^*, y^*)\) can be attained by the tariff vector \((\tau_0, \tau_1, \tau_2) = (\tau_0, f^1(\tau_0), f^2(\tau_0))\), where

\[
f^1(\tau_0) \equiv 1 - (1 - \tau_0) \frac{q_0^* p_1}{q_1^*}, \quad f^2(\tau_0) \equiv 1 - (1 - \tau_0) \frac{q_0^* p_2}{q_1^*}.
\] (12)

**Proof.** By letting \(\tau_0 = \tau_0^*\), the first equation in (7) yields the implicit level of \(\kappa\) for \(\tau_0\), which is \(\kappa = 1/((1 - \tau_0)q_0^*)\). Substituting this for \(\kappa\) of the second and third equations in (7) yields (12). Q.E.D.

From Proposition 1, the tariff vector \((\tau_0, \tau_1, \tau_2)\) that supports a particular equilibrium allocation of the GTE for a given tariff revenue is not unique. However, when one of the three tariff rates is fixed at a given level, the levels of the other two tariff rates are determined.

As a special case of the GTE, consider an economy satisfying \(\tau_0 = 0\). Then, from (7),
the corresponding tariff vector is given by

\[
(0, f^1(0), f^2(0)) = \left(0, 1 - \frac{q_0^* p_i}{q_i^*}, 1 - \frac{q_0^* p_2}{q_i^*}\right).
\]  

(13)

The following lemma is used in subsequent analysis.

**Lemma 2.** Then functions \(f^1(\tau_0)\) and \(f^2(\tau_0)\) defined in Lemma 1 can be alternatively expressed as

\[
f^1(\tau_0) = (1 - \tau_0)f^1(0) + \tau_0, \quad f^2(\tau_0) = (1 - \tau_0)f^2(0) + \tau_0.
\]  

(14)

**Proof.** From (12) we have

\[
\frac{q_0^* p_i}{q_i^*} = 1 - f'(0).
\]

Applying this to (12), we obtain

\[
f'(\tau_0) = 1 - (1 - \tau_0)(1 - f'(0)) = (1 - \tau_0)f'(0) + \tau_0, \quad i = 1, 2.
\]  

Q.E.D.

**b. Import tariff rates as a function of the duty drawback rate \(\beta\) in the DDE**

Define the tariff vector \((\tau_0, \tau_1, \tau_2)\) of the DDE by (6) and the duty-draw-back-tariff vector \((\beta, \tau_1, \tau_2)\) by (11).

Since Lemma 2 is satisfied in the DDE, we have the following:

**Lemma 3.** Let \((q_0^*, q_1^*, q_2^*)\) be an equilibrium price vector and \((x^*, y^*)\) be the equilibrium allocation achieved at \((q_0^*, q_1^*, q_2^*)\) of the DDE. If \((\beta, \tau_1, \tau_2)\) is an equilibrium duty-draw-back-tariff vector that supports this equilibrium. The import tariff rates \(\tau_1, \tau_2\) can be rewritten as functions of \(\beta\) and in terms of the functions of \(f'(0)\) defined in Lemma 2 as

\[
\tau_1 = (1 - \alpha \beta \tau_2)f^1(0) + \alpha \beta \tau_2, \quad \tau_2 = (1 - \alpha \beta \tau_2)f^2(0) + \alpha \beta \tau_2
\]

(15)
Proof. Substituting (11) for $\tau_0$ in (14), we immediately obtain the lemma. Q.E.D.

c. Optimal import tariff rates as a function of the duty drawback rate $\beta$ in the DDE

When one element of $(\beta, \tau_1, \tau_2)$ is fixed at a level, the optimal levels of the remaining elements (policy variables) are uniquely determined. Let us now examine the relationship between $\beta$ and the optimal import tariffs. Let $\xi_1$ and $\xi_2$ be the optimal tariff of the DDE when $\beta = 0$. Substituting $\xi_i$ for $f'(0)$ in (15), and solving for $\tau_1$ and $\tau_2$ yields the following:

**Proposition 6.** The optimal import tariffs, $\tau_1$ and $\tau_2$ in the DDE for a value of $\beta$ can be expressed as

$$
\tau_1(\beta) = \frac{(1-\alpha\beta)\xi_1 + \alpha\beta\xi_2}{(1-\alpha\beta) + \alpha\beta\xi_2}, \quad \tau_2(\beta) = \frac{\xi_2}{(1-\alpha\beta) + \alpha\beta\xi_2}.
$$

(16)

Proposition 6 implies the following:

**Proposition 7.** Assume that $\xi_1 > 0$ and $\xi_2 > 0$. Then the following holds in the DDE.

(i) The optimal import tariffs $\tau_1$ and $\tau_2$ are always positive for any value of $\beta$.

(ii) As $\beta$ increases, the optimal tariff rates on goods 1 and 2 increase.

Proof. Given $0 < \alpha < 1$ and $0 \leq \beta \leq 1$, we have $0 \leq \alpha\beta < 1$. This and (16) immediately prove (i). Next, we prove (ii). Given $\xi_1$ and $\xi_2$ are constant, (16) yields

$$
\frac{d\tau_1}{d\beta} = \frac{\alpha\xi_2(1-\xi_1)}{[1+\alpha\beta(\xi_2 - 1)]^2}, \quad \frac{d\tau_2}{d\beta} = \frac{\alpha\xi_2(1-\xi_2)}{[1+\alpha\beta(\xi_2 - 1)]^2}.
$$

(17)

As $0 < \xi_i < 1$ from the assumption and definition of the tariff rate, $d\tau_i/d\beta > 0$. Q.E.D.

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9 Section 5 shows that the optimal tariffs on the imported good and the imported input are both positive if all goods (including the imported input) are substitutable. Note that Lopez and Panagariya (1992) considered three situations where the imported input is always complementary with one final good. In contrast, Hatta and Ogawa (2003) provided the situations where all goods including the imported input are substitutable.
Figure 1 depicts the graphs of (16) in the case of \( \xi_1 > \xi_2 \) for a given \( \alpha \), given values of \( \xi_1 \) and \( \xi_2 \), and for various values of \( \beta \) satisfying (16). Because \( \alpha < 1 \) holds, a full duty drawback (\( \beta = 1 \)) is allowed. If \( \beta \) exceeds one and approaches \( 1/\alpha \), the optimal tariff rates on goods 1 and 2 approach one. Substituting \( \beta = 1/\alpha \) into (16) yields \( \tau_1 = \tau_2 = 1 \). However, the optimal tariff rates are never equal to one from the definition of the ad valorem tariff rate. Figure 1 shows that the optimal tariff on good 2 is always positive in the range of \( \beta < 1/\alpha \) and that \( \beta \) is negative when \( \tau_1 < \xi_1 \).

5. Optimal tariff of the DDE when \( \beta = 0 \)

In Proposition 6, we expressed the optimal import tariffs of the DDE as functions of \( \xi_1 \) and \( \xi_2 \), the optimal tariffs when \( \beta = 0 \). In the present section, we in turn express \( \xi_1 \) and \( \xi_2 \) as functions of the elasticities of excess demand functions as follows:

**Proposition 8.** The optimal tariff rates for the two imports of the DDE when \( \beta = 0 \) are given by

\[
\xi_1 = (\eta_{20} + \eta_{12} + \eta_{21})\theta, \quad (18a)
\]

\[
\xi_2 = (\eta_{00} + \eta_{12} + \eta_{21})\theta, \quad (18b)
\]

where \( \eta_{ij} = (\partial z_i / \partial q_j) \cdot (q_i / z_j) \) is the elasticity of the excess demand for the \( i \)-th good with respect to the \( j \)-th good and \( \theta \) is a positive scalar.

**Proof.** The duty-drawback-tariff combination of a DDE can be expressed by the tariff

\[
\tau_1 = \frac{(1 - \alpha \beta)(\xi_1 - \xi_2)}{(1 - \alpha \beta) + \alpha \beta \xi_2},
\]

which shows that \( \tau_1 > (\alpha, <) \tau_2 \) if \( \xi_1 > (\alpha, <) \xi_2 \).
vector of the corresponding GTE. Thus the optimal import tariff rates $\xi_1^\tau$ and $\xi_2^\tau$ of the GTE with $\tau = 0$ are equal to the optimal import tariff rates $\xi_1$ and $\xi_2$ of the corresponding DDE with $\beta = 0$. Thus we have

$$\xi_i^\tau = \xi_i, i = 1, 2. \quad (19)$$

Lemma (iii) in Hatta and Ogawa (2003, p. 12) provided the formula of the optimal import tariff rates of the GTE when $\tau = 0$. In terms of this formula and (19), we have Proposition 8. Q.E.D.

If all three goods of the DDE are substitutable for each other, i.e., if the excess demand elasticities in (10) are all positive, therefore, the optimal tariffs on the import good and imported input are both positive$^{11}$, and the premise of Proposition 7 is satisfied.

There are two cases where this premise is not satisfied.

The first case is when the imported input is complementary with the export good. Then $\eta_{20} < 0$ holds, and equation (19a) implies that optimal tariff on the imported final good $\xi_1$ can be negative. However, equations (19b) implies that the optimal tariff on the imported input $\xi_2$ is necessarily positive then, since the imported input cannot then be complementary with the imported final good and $\eta_{21} > 0$ holds$^{12}$.

The second case is when the imported input is complementary with the imported final good. Then, $\eta_{12} + \eta_{21} < 0$ holds and either $\xi_1$ or $\xi_2$ can be negative from (18)$^{13}$. In that case, the imported input needs be subsidized even when there is no duty drawback.

Note that by substituting (18) for $\xi_1$ or $\xi_2$ in (16), we obtain precise formula for import tariffs for any non-zero value of $\beta$.

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$^{11}$ Note that from the assumption (9), the elasticities related to the imported input can now be expressed by $\eta_{20} = y_{20}/y_2$, $\eta_{21} = q_{i1}y_{21}/y_2$, and $\eta_{12} = -q_{21}y_{12}/y_1$.

$^{12}$ Since the DDE is a three-good economy, when the imported input is complementary with the export good, it must be substitutable for the imported final good.

$^{13}$ Lopez and Panagariya (1992) analyzed the three situations where the imported input is always complementary with one final good because of the Leontief type technology. On the other hand, Hatta and Ogawa (2003) studied the situations where all goods including the imported input are substitutable for each other.
6. Conclusion

We analyzed revenue-constrained optimal tariffs in the presence of a duty drawback on the export good. Our findings are as follows. First, the combination of an import tariff and duty drawback can be at a level that is most suitable politically because the optimal combination is not unique. Second, if optimal import tariffs are positive when the duty drawback is zero, then the import tariffs are positive for all positive values of the duty drawback.
References


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\( \tau_1 = \tau_1(\beta) \)
\( \tau_2 = \tau_2(\beta) \)

Full duty drawback

Figure 1