Irregular Economic Growth in the World Economy: Fluctuations of Ergodic Distributions through a Markov Chain Model

Alvaro Domínguez
Asian Growth Research Institute (AGI)

Hiroshi Sakamoto
Asian Growth Research Institute (AGI)

Working Paper Series Vol. 2022-03

February 2022

Asian Growth Research Institute

Irregular Economic Growth in the World Economy: Fluctuations of Ergodic Distributions through a Markov Chain Model

Alvaro Domínguez[1]
Asian Growth Research Institute (AGI)
11-4 Otemachi, Kokurakita, Kitakyushu, 803-0814 JAPAN

Hiroshi Sakamoto[2]
Asian Growth Research Institute (AGI)
11-4 Otemachi, Kokurakita, Kitakyushu, 803-0814 JAPAN

Abstract

We reexamine the convergence hypothesis of economic growth. Traditionally, it was analyzed using econometric methods, although estimating long-term economic fluctuations with a linear model is not always ideal. We thus employ a Markov chain stochastic model that divides the logarithmic value of relative income, comparing each country's GDP per capita with the average, into several ranks in descending order of income. Using the most recent data, we total the time-series changes of the income states in each sample, and represent them through probabilities. We observe the changing ergodic distribution and show that the world economy is not growing monotonously, and proceed to correct the population size of each country for rank changes. The transition probability matrix is re-estimated by applying population weights to changes in the income states of each country. When there is no population weighting, the model shows that the world economy may be divided into two peaks as before. However, when using population weights, the model yields more optimistic results.

JEL classification: C49, D39, O50, R11
Keywords: Convergence, World Economy, Markov Chain

[1] *E-mail address: alvdom@agi.or.jp*

[2] *E-mail address: sakamoto@agi.or.jp*

1. Introduction

We analyze the long-term growth trends of the world economy from the 1950s, from the perspective of convergence. We study whether economically poor countries (regions) can catch up with rich ones. Economic theory tells us that if we assume production functions with diminishing returns à la Solow (1956), the possibility of convergence increases. This is due to productivity being high at the stage of low input and gradually decreasing as inputs increase. By replacing “low input” with “poor country (region)”, we can then link it to empirical research.

There is a large literature related to whether the world economy is converging (Barro and Sala-i-martin, 2004; Islam, 2003). One finding is that there is a lack of convergence due to the economic growth of each country being non-monotonous. Various economic growth models such as Solow (1956) emphasize monotony as a characteristic. However, the economic growth of each country does not behave exactly as in a model.

This discussion has traditionally been analyzed mainly through econometric methods. In this study, however, we rely on the stochastic method proposed by Quah (1993, 1996a, 1996b). This unique analytical method, classified by Islam (2003) as a “distribution approach,” produces distinctive results compared to econometric methods. It divides the logarithm of relative income, which compares each country's GDP per capita with the average, into several ranks in descending order of income. Then, the time-series changes of the rank (income states) in each sample are totaled, and these are represented as probabilities. The Markov transition matrix obtained by such a procedure has an eigenvalue (the “dominant eigenvalue”) whose absolute value is 1. Therefore, the convergence distribution (ergodic distribution) can be estimated by obtaining the associated eigenvector of the eigenvalue taking a value of 1. With this approach, Quah argues that the world economy is divided into two extremes or “Twin Peaks”: a rich economy and a poor economy.

In this paper, we analyze how the ergodic distribution obtained by the above procedure varies over time. This shows a well-known tendency in econometrics that changing the sample changes the result, but conversely, the data does not change monotonously. In other words, trends in the world economy change in a complex way.

In previous studies, the changes in income state were calculated using a country (region) as a sample. This has the problem that it is not easy for the income state to change in countries with large populations such as China and India. Hence, in this work, we multiply each sample by the population weight and calculate the Markov transition matrix from the aggregated results to obtain the convergence distribution. In the case of the population weight, the average of the year in which the sample as the number of populations appears and the year after it was calculated, the ratio is obtained from the total of these worlds, and is multiplied by the income states movement. This makes the stochastic model more precise, although the aggregation work becomes more complex because the population ratio has to be multiplied for all state changes.

This work is organized as follows. In section 2 we describe the stochastic model using

Markov chains. Section 3 presents the data we use. We show the results of the convergence distribution without population weight in section 4 and with population weight in section 5. Section 6 concludes.

2. The Stochastic Model

The "stochastic model" using a Markov transition matrix is defined as follows. Let F_t be the income distribution state in period *t*, expressed as a $1\times N$ vector, representing the number of states. The Markov process is an expression of the situation in which the income distribution situation F_{t+1} in the next state depends on the income distribution state F_t in the current state. In other words, the fluctuation of the income distribution states between the two time points is defined as follows.

$$F_{t+1} = F_t \cdot M_t \quad (1)$$

Where M_t is a transition matrix. Now, since the income distribution is ranked for this transition, the transition matrix is estimated by aggregating the changes of income states. The estimation method is as follows, with *P* defined as the number of changes of income states.

$$M_{t,jk} = P_{t,jk} / \sum_{k=1}^{n} P_{t,jk} \quad (2)$$

Next, when population weighting is applied to each transition, the transition probability matrix is estimated after totaling the population weighting *W* that differs in each country (region).

$$M_{t,jk} = \sum_{r=1}^{m} W_{t,jk,r} / \sum_{k=1}^{n} \sum_{r=1}^{m} W_{t,jk,r} \quad (2)'$$

This shows the probability of how much of the total number of changes (total number of income states movements from state *k*), including the number staying in the same state has moved to state *j*. Then, we take advantage of the ergodicity which is a characteristic of Markov chains, and find the convergence distribution (ergodic distribution).

$$F = F \cdot M \quad (3)$$

The transition matrix obtained through such a procedure usually has an eigenvalue with an absolute value of 1. Therefore, by finding the eigenvector associated to this eigenvalue, the estimation of the convergence or ergodic distribution can be done by using a solver such as the one found in Excel. However, in this study, we make use of GAMS (General Algebraic Modeling System) to calculate various convergence distributions. In this case, equation 3 holds

when *M* is given, and the model is constrained so that the sum of *F* equal to 1 is solved.

The convergence distribution analyzed in this study consists of a fixed distribution that is reached through a stochastic model composed of samples from 1953 to 2017. The convergence hypothesis examines whether indicators such as GDP per capita for the world economy or a specific economic group settle into a certain value. When verifying whether there is convergence through this model, the shape of the convergence distribution is a concern. Hence, assuming that there is convergence to a certain value, it is desirable that the distribution is concentrated in some income states. Particularly, it is expected that the distribution will be mountain-shaped, centered on the middle-income group. For other distributions instead, convergence may not be achieved.

3. Data

For the data in this study, we rely on the PWT (Penn World Table version 9.1, https://www.rug.nl/ggdc/productivity/pwt/), which provides internationally comparable data (Feenstra, Inklaar and Timmer, 2015). The PWT provides only macroeconomic indicators such as GDP, but for some countries, it is superior because it provides estimates from 1950. However, since the data for China is available only from 1952 onwards, we analyze from 1952 to 2017 (the last obtainable year). The number of country (region) changes during the period, is set to a maximum of 182.

Next, we use real GDP and population. The PWT estimates real GDP from two sides, production and expenditure, with the estimated values being slightly different. In this work, we utilize real GDP ("rgdpo" in the PWT notation) from the production side. Then, from this data we calculate the relative income per capita. The relative income is defined as the logarithm when the average income (of each year) is 1 ($ln(y/n/\sum y/\sum n)$), with *y* representing the real GDP, and *n* being the population.

The analysis of convergence in this paper shows fluctuations in the relative income of each country (region) through a stochastic model. Specifically, we rely on the method proposed by Quah (1993, 1996a, 1996b). In this method, the logarithmic value of the relative income mentioned above is ranked in ascending order of income, with the change in rank being followed in chronological order, and the aggregated value being expressed as a probability. However, since this ranking is arbitrary, we fix the logarithmic value of the relative income for the ranking, with the following numbers based on the average of 0:

5 states: (lo, ml, mi, mh, hi or 1, 2, 3, 4, 5).

-1.386 (1/4), -0.693 (1/2), 0.000 (1), 0.693 (2)

7 states: (ll, lo, ml, mi, mh, hi, hh or 1, 2, 3, 4, 5, 6, 7).

-1.500, -1.000, -0.500, 0.000, 0.500, 1.000

The numbers in parentheses in the five states indicate 1/4, 1/2, and 2 times the average income. This makes it possible to respond to changes in the number of samples.

4. Time Series Change of Convergence Distribution without Population Weight

Up to this point, we have observed the convergence distribution through the stochastic model by region and confirmed the distribution of the income groups that make up this model. Next, we analyze the changes in the time series with respect to the convergence of the world economy. In this study, the purpose of fixing the relative income (i.e., the standard of the income class), is that it is possible to deal with changes in the number of samples. This allows us to analyze the difference in the convergence distribution when the measurement period is changed. Thus, we start by calculating the convergence distribution using the sample going from 1953 to 1972 for each income state. We then extend the convergence distribution period, on a yearly basis (e.g., 1973, 1974, and so on), by computing the cumulative type samples. Additionally, we compute the convergence distribution through the moving average type samples for 20 years (e.g., 1954-73, 1955-74, etc.), and we then compare it to the cumulative type samples obtained previously.

Figure 1 shows the convergence distribution of five income states from the cumulative sample in chronological order. From this figure, we see that the convergence distribution of the 1953-72 sample converges to the high-income class, and then it gets polarized over time. In table 1, the distribution of the high-income group is the largest throughout the entire period, but the second-largest group changes to the low-income group. This indicates that the convergence distribution changes depending on the sample used, and that it does not become a universal event for future trends in the world economy.

Figure 2 shows the convergence distribution of the five income states of the cumulative moving average sample in chronological order. First, in the early stages, the distribution is concentrated in the high-income group, and then from around the 1980s, it tends to become polarized. From around 1994, it becomes a bipolar differentiation concentrated in the low-income group. Then, approximately from 2010, it becomes a distribution concentrated in the middle-income group. From this result, the convergence distribution tends to change significantly when the period is divided.

The results of these calculations, in seven income states, are shown in Figure 3 (cumulative sample) and Figure 4 (moving average sample). There is a slight difference in the year of change depending on the moving average type of the sample, but the tendency is not much different from the case of five income states. There is no universality in the future trends of the world economy, and it becomes clear that the convergence problem has not yet been solved.

5. Time Series Change of Convergence Distribution with Population Weight

Figure 5 shows the convergence distribution of five income states through the cumulative sample in chronological order. We can see from this figure that the overwhelming concentration of distribution in the high-income group tends to gradually disappear. However, from 2000 to 2002 the tendency to concentrate once again returns, and since then it gradually eases. Nevertheless, it shows a high ratio of 60% or more throughout the measurement period, indicating that the world economy is likely to be concentrated in the high-income group. The group with the lowest ratio is the middle-income group until around 2005, but after 2006 it becomes the low-income group. Accordingly, up to 2005 we can say that the differentiation is very weak, even though it is concentrated in the high-income group.

Figure 6 shows the convergence distribution of five income states from the moving average type of the cumulative sample in chronological order. In the figure, we can observe the color changes. Instead, in table 6 the concentration on the high-income group continues until 1990. It should be noted though, that the ratio drops significantly. Additionally, from 1991 the hierarchy of the highest ratio changes rapidly. In section 4 we pointed out that the convergence distribution tends to change significantly when the period is divided, but it is safe to say that a similar tendency occurs even if the population weight is considered.

The results of these calculations into seven income states are shown in Figure 7 (cumulative sample) and Figure 8 (moving average sample). We can see that the highest-income group has the highest ratio in the cumulative sample, although the ratio gradually decreases. There is a weak bipolar differentiation between 1992 and 2008, but it is concentrated in the second lowest income group (*lo*) rather than the lowest income one (*ll*). For the moving average type of sample, the same concentration on the highest income group (as in the five states) continues until 1989, whereas in the concentration on the lowest income group persists from 1990 up to 2001. From then on, with some exceptions, the distribution concentrates in the middle-income class. Although there are some differences, these do not considerably differ with those from the tendency in the five states. In addition, the possibility of bipolar differentiation disappears when we consider population weighting. We can confirm that the “Twin Peaks” claimed by Quah are not found.

Finally, regarding the convergence hypothesis, the concentration of the distribution in the high-income states can initially be seen as a confirmation of the hypothesis. However, in this study it is difficult to say that it is achieved in two different ways. In the first, if strictly attained, the income distribution should be more average, in which case the distribution ratio of the average income group would be the highest. In the second, although the distribution is concentrated in the high-income group, the ratio declines. Due to this, the absolute convergence hypothesis cannot be obtained, and we conclude that instead conditional convergence occurs. This is similar to the case of many studies relying on econometric methods.

6. Summary

We considered the traditional convergence hypothesis discussed in economics as a way to analyze the inequality distribution affecting various countries. We then verified this relying on data from the Penn World Table. After introducing a Quah-style Markov chain model, we presented results while considering populations with and without weights. We showed that there is a tendency for a bipolar differentiation of the world economy, whereas the convergence hypothesis does not hold. Meanwhile, the distribution tends to be concentrated in the higher income group when considering population weighting. Possibly, this tendency differs depending on the time under analysis, and does not necessarily lead to the establishment of the hypothesis. Nonetheless, since the growth of the world economy is not considered to be uniform, it is an expected result.

References

Barro, Robert J. and Xavier Sala-i-Martin. (2004) *Economic growth (Second edition)*, Cambridge: MIT Press

Feenstra R., Inklaar. R, and Timmer. M, (2015) "The Next Generation of the Penn World Table," *American Economic Review*, 105(10), pp. 3150-3182.

Islam, Nazrul. (2003) "What Have We Learnt from the Convergence Debate? A Review of the Convergence Literature," *Journal of Economic Surveys*, 17(3), pp. 309-362.

Quah, Danny. (1993) "Empirical Cross-Section Dynamics in Economic Growth," *European Economic Review*, 37, pp. 426-434.

Quah, Danny. (1996a) "Empirics for Economic Growth and Convergence," *European Economic Review*, 40, pp. 1353-1375.

Quah, Danny. (1996b) "Twin Peaks: Growth and Convergence in Model of Distribution Dynamics," *Economic Journal*, 106, pp. 1045-1055.

Solow, Robert M. (1956) "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70, pp. 65-94.

Figure 1 Five States Ergodic Distribution without Population Weight (Accumulation from 1972)

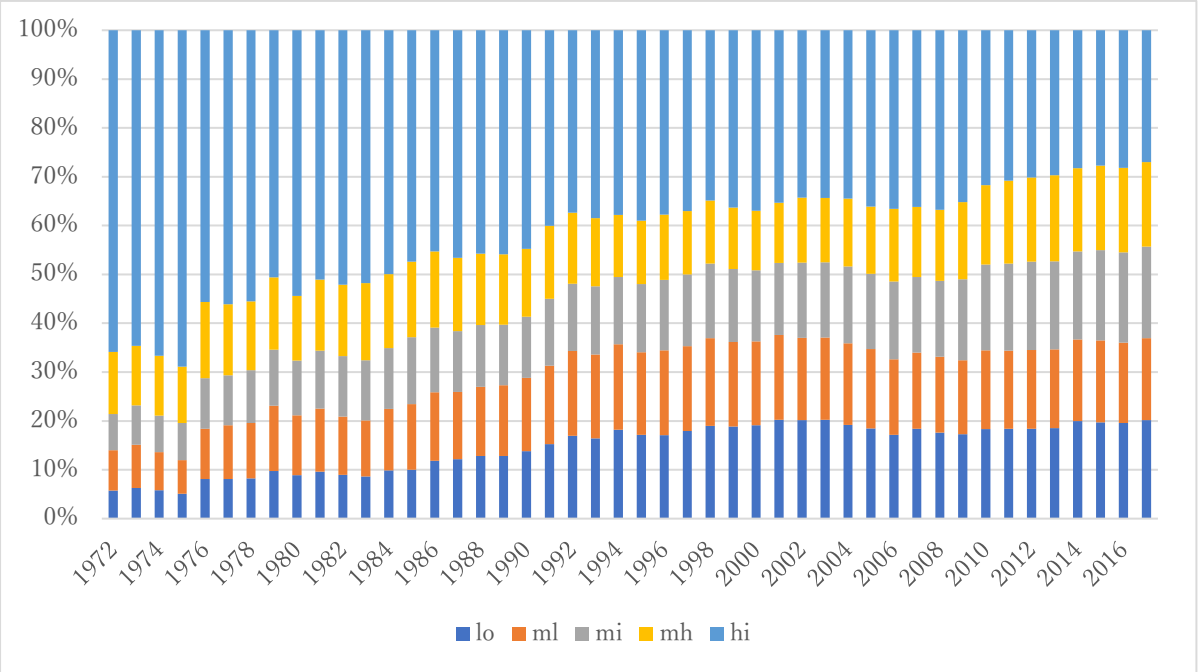


Figure 2 Five States Ergodic Distribution without Population Weight (20 Years Moving Average)

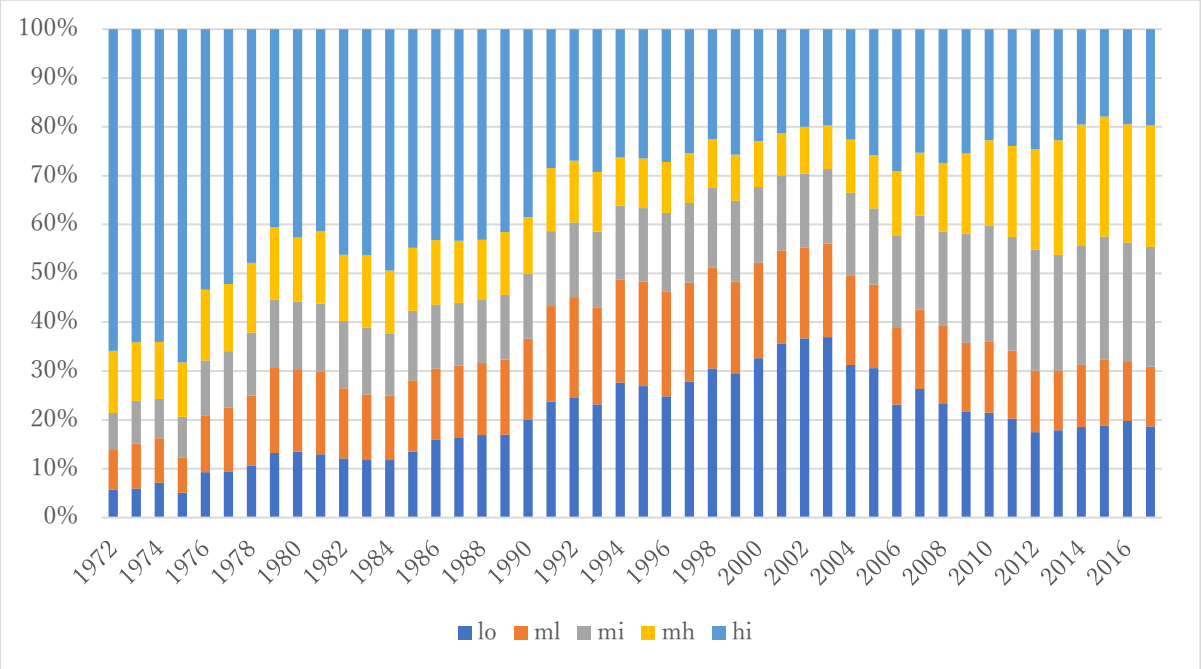


Figure 3 Seven States Ergodic Distribution without Population Weight (Accumulation from 1972)

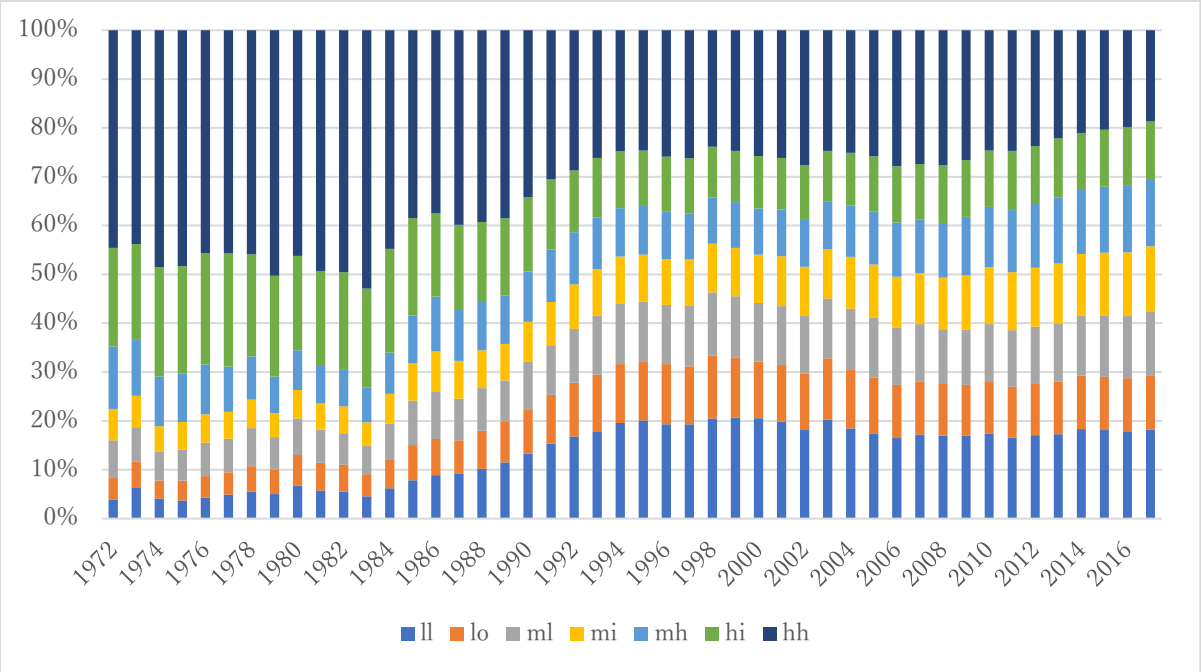


Figure 4 Seven States Ergodic Distribution without Population Weight (20 Years Moving Average)

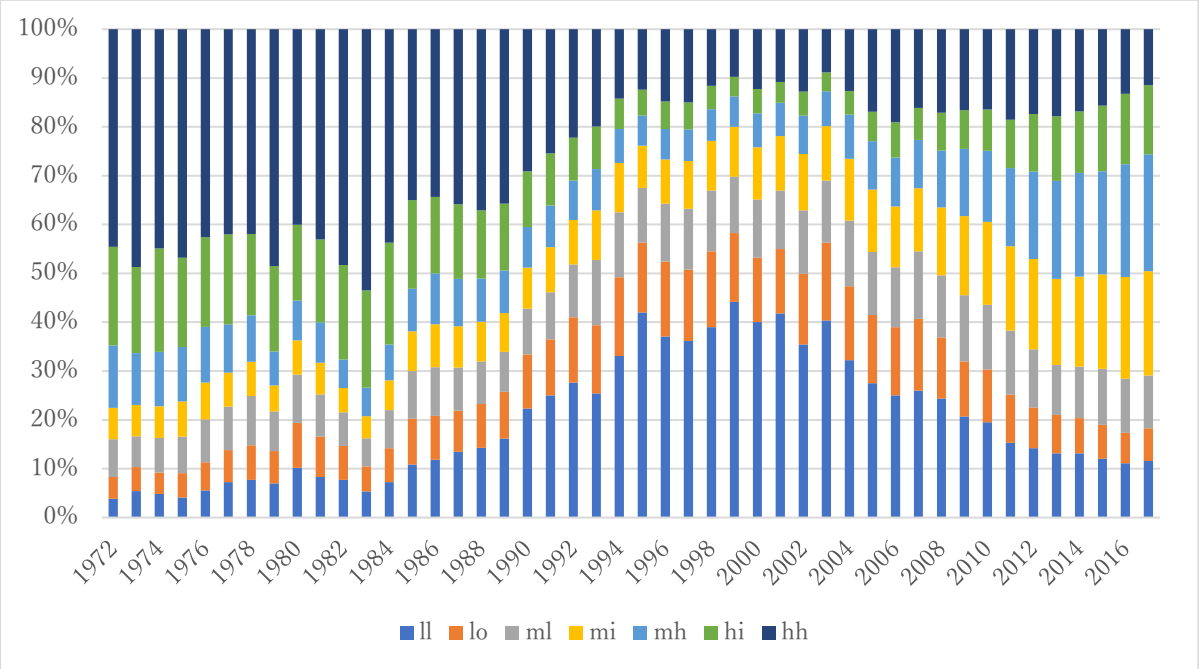


Figure 5 Five States Ergodic Distribution with Population Weight (Accumulation from 1972)

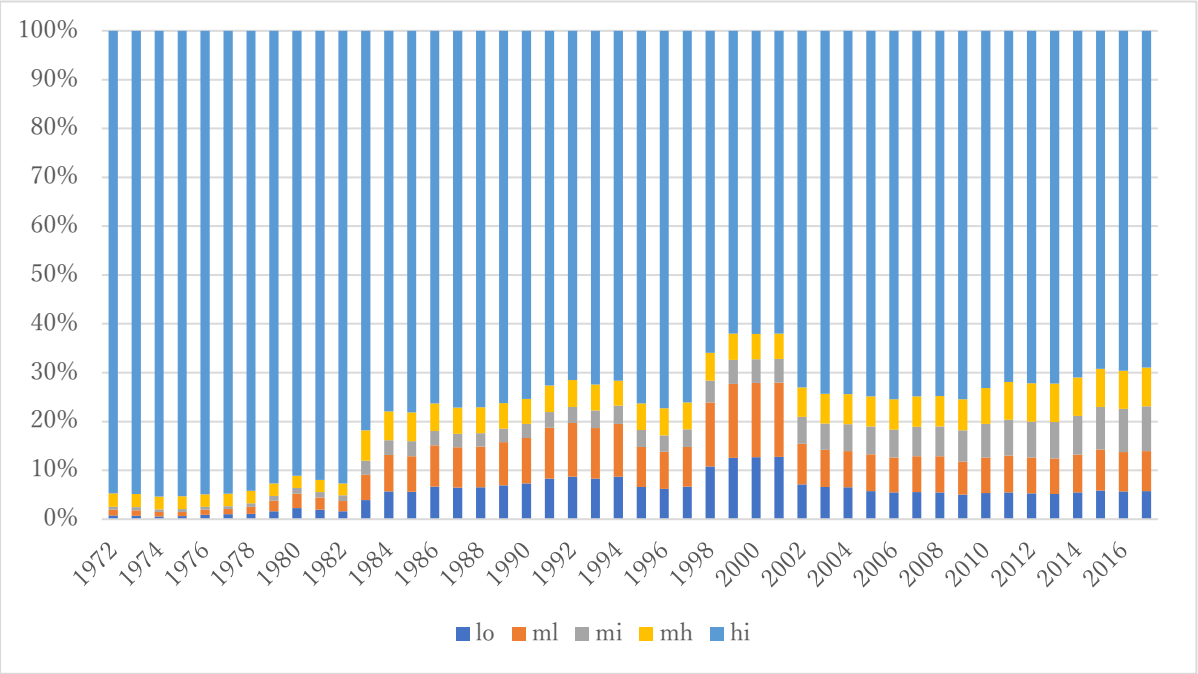


Figure 6 Five States Ergodic Distribution with Population Weight (20 Years Moving Average)

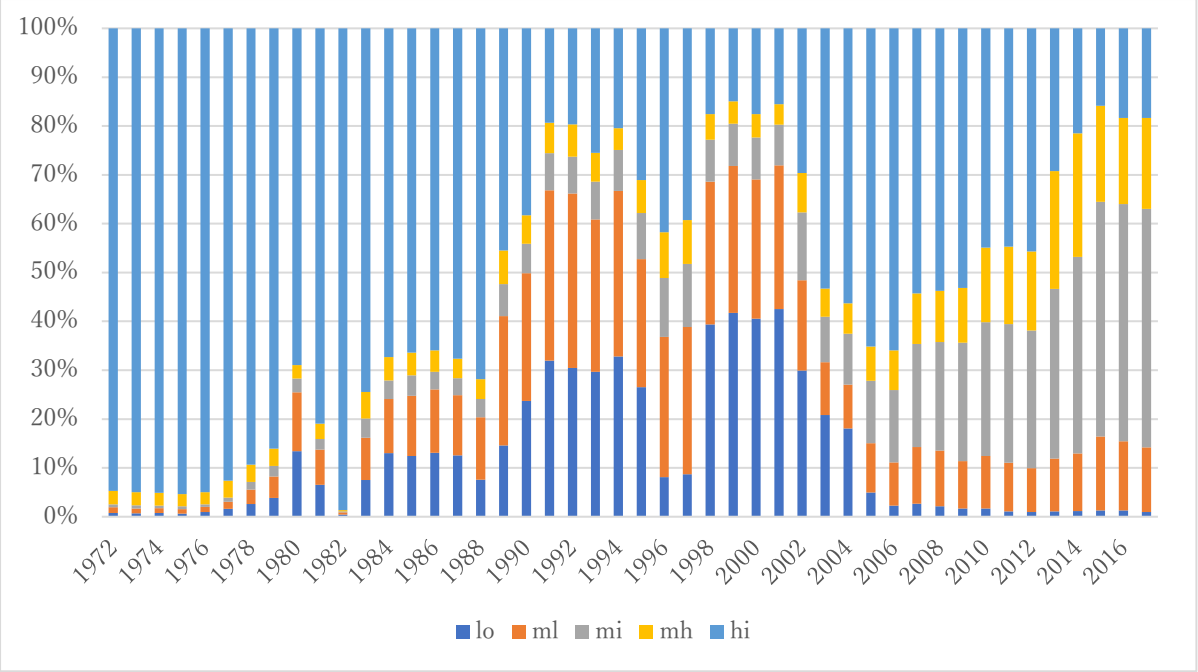


Figure 7 Seven States Ergodic Distribution with Population Weight (Accumulation from 1972)

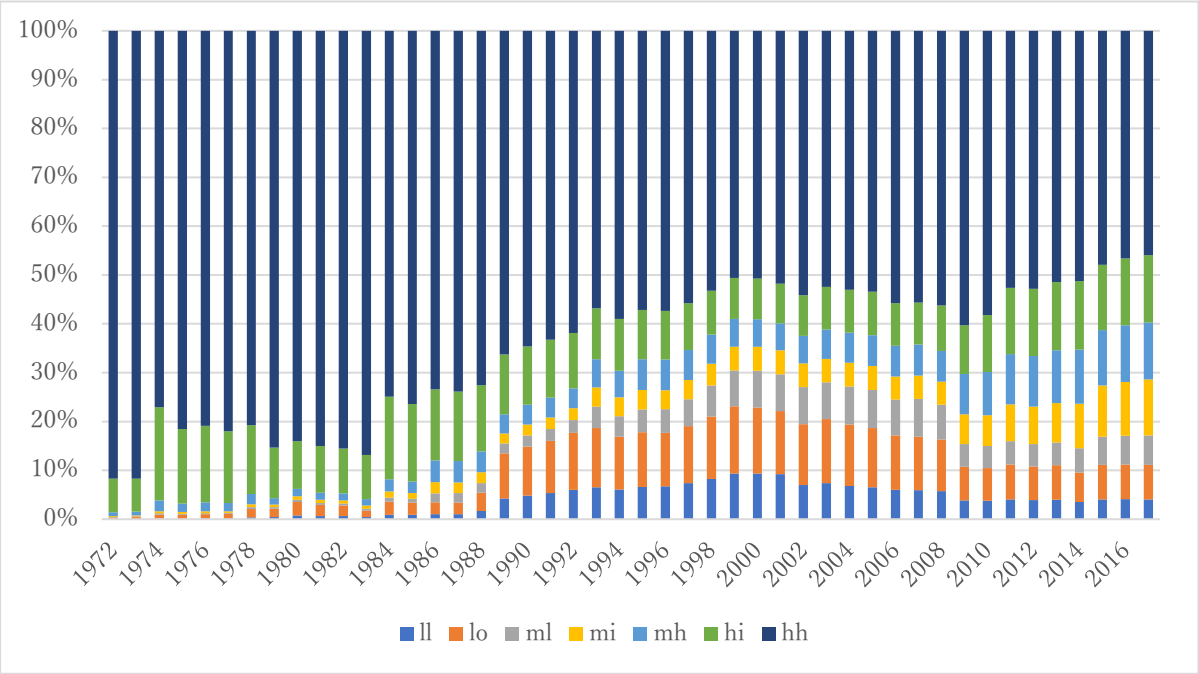
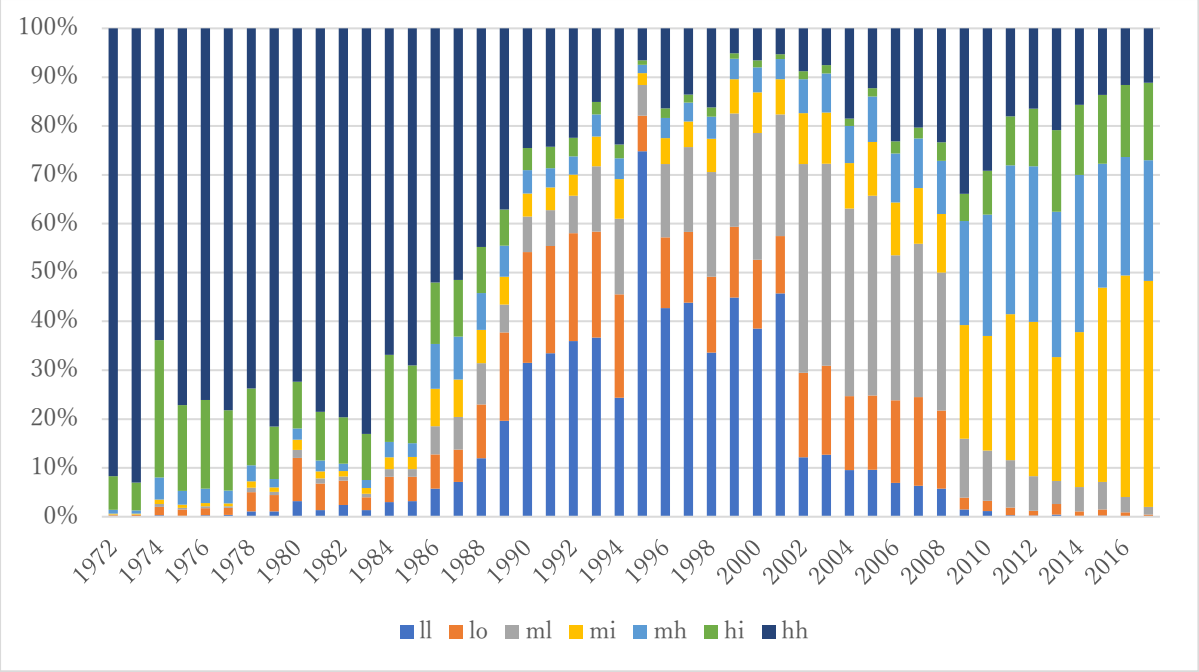


Figure 8 Five States Ergodic Distribution with Population Weight (20 Years Moving Average)



Appendix Table 1 Five States Ergodic Distribution without Population Weight (Accumulation from 1972)

	lo	ml	mi	mh	hi
1972	0.0575	0.0825	0.0744	0.1269	0.6587
1973	0.0625	0.0890	0.0803	0.1217	0.6465
1974	0.0581	0.0780	0.0748	0.1227	0.6663
1975	0.0512	0.0682	0.0765	0.1149	0.6891
1976	0.0810	0.1027	0.1040	0.1562	0.5561
1977	0.0809	0.1105	0.1023	0.1451	0.5611
1978	0.0827	0.1129	0.1085	0.1411	0.5548
1979	0.0978	0.1334	0.1147	0.1482	0.5059
1980	0.0892	0.1220	0.1122	0.1324	0.5442
1981	0.0965	0.1287	0.1185	0.1455	0.5108
1982	0.0896	0.1193	0.1240	0.1460	0.5210
1983	0.0866	0.1145	0.1231	0.1582	0.5177
1984	0.0985	0.1259	0.1247	0.1512	0.4997
1985	0.0999	0.1346	0.1369	0.1545	0.4741
1986	0.1187	0.1401	0.1321	0.1565	0.4526
1987	0.1215	0.1381	0.1240	0.1504	0.4660
1988	0.1283	0.1416	0.1267	0.1458	0.4576
1989	0.1285	0.1449	0.1238	0.1442	0.4586
1990	0.1380	0.1505	0.1252	0.1390	0.4474
1991	0.1524	0.1607	0.1372	0.1491	0.4006
1992	0.1693	0.1740	0.1377	0.1453	0.3738
1993	0.1641	0.1720	0.1398	0.1393	0.3848
1994	0.1820	0.1750	0.1379	0.1272	0.3779
1995	0.1718	0.1690	0.1394	0.1296	0.3902
1996	0.1708	0.1735	0.1448	0.1335	0.3774
1997	0.1792	0.1739	0.1467	0.1298	0.3704
1998	0.1902	0.1791	0.1528	0.1295	0.3485
1999	0.1885	0.1732	0.1497	0.1254	0.3632
2000	0.1913	0.1717	0.1454	0.1222	0.3693
2001	0.2024	0.1736	0.1479	0.1231	0.3531
2002	0.2015	0.1683	0.1543	0.1330	0.3428
2003	0.2024	0.1685	0.1540	0.1321	0.3431
2004	0.1922	0.1669	0.1573	0.1390	0.3447
2005	0.1845	0.1625	0.1542	0.1376	0.3612
2006	0.1715	0.1550	0.1588	0.1487	0.3659
2007	0.1838	0.1560	0.1548	0.1434	0.3621
2008	0.1764	0.1554	0.1551	0.1453	0.3678
2009	0.1729	0.1511	0.1662	0.1578	0.3520
2010	0.1832	0.1614	0.1756	0.1629	0.3170
2011	0.1838	0.1604	0.1783	0.1696	0.3079
2012	0.1839	0.1614	0.1811	0.1724	0.3012
2013	0.1852	0.1612	0.1802	0.1766	0.2968
2014	0.2000	0.1666	0.1806	0.1701	0.2827
2015	0.1970	0.1677	0.1851	0.1730	0.2771
2016	0.1960	0.1644	0.1846	0.1732	0.2819
2017	0.2015	0.1678	0.1879	0.1731	0.2697

Appendix Table 2 Five States Ergodic Distribution without Population Weight (20 Years Moving Average)

	lo	ml	mi	mh	hi
1972	0.0575	0.0825	0.0744	0.1269	0.6587
1973	0.0597	0.0921	0.0870	0.1203	0.6410
1974	0.0712	0.0914	0.0807	0.1164	0.6403
1975	0.0511	0.0710	0.0844	0.1110	0.6825
1976	0.0929	0.1151	0.1138	0.1444	0.5338
1977	0.0935	0.1318	0.1155	0.1373	0.5218
1978	0.1067	0.1427	0.1293	0.1428	0.4784
1979	0.1320	0.1756	0.1385	0.1482	0.4056
1980	0.1350	0.1691	0.1380	0.1315	0.4264
1981	0.1294	0.1694	0.1394	0.1485	0.4133
1982	0.1210	0.1436	0.1379	0.1357	0.4618
1983	0.1175	0.1350	0.1363	0.1485	0.4627
1984	0.1175	0.1322	0.1262	0.1298	0.4943
1985	0.1347	0.1467	0.1417	0.1292	0.4477
1986	0.1592	0.1459	0.1304	0.1330	0.4316
1987	0.1646	0.1468	0.1283	0.1271	0.4331
1988	0.1692	0.1470	0.1296	0.1231	0.4311
1989	0.1695	0.1541	0.1327	0.1282	0.4156
1990	0.2013	0.1648	0.1328	0.1164	0.3847
1991	0.2371	0.1957	0.1540	0.1285	0.2847
1992	0.2460	0.2052	0.1532	0.1262	0.2693
1993	0.2319	0.1988	0.1545	0.1224	0.2924
1994	0.2763	0.2105	0.1517	0.0985	0.2629
1995	0.2695	0.2139	0.1504	0.1018	0.2644
1996	0.2480	0.2150	0.1607	0.1045	0.2719
1997	0.2783	0.2032	0.1628	0.1020	0.2537
1998	0.3054	0.2070	0.1636	0.0984	0.2257
1999	0.2955	0.1879	0.1652	0.0945	0.2568
2000	0.3261	0.1954	0.1555	0.0936	0.2293
2001	0.3556	0.1916	0.1535	0.0862	0.2131
2002	0.3672	0.1854	0.1514	0.0959	0.2001
2003	0.3694	0.1915	0.1530	0.0887	0.1974
2004	0.3125	0.1825	0.1699	0.1092	0.2260
2005	0.3060	0.1709	0.1562	0.1085	0.2584
2006	0.2310	0.1586	0.1869	0.1323	0.2912
2007	0.2635	0.1627	0.1918	0.1291	0.2529
2008	0.2323	0.1602	0.1926	0.1408	0.2742
2009	0.2172	0.1406	0.2233	0.1648	0.2542
2010	0.2150	0.1457	0.2365	0.1753	0.2275
2011	0.2023	0.1395	0.2328	0.1859	0.2394
2012	0.1745	0.1261	0.2477	0.2060	0.2456
2013	0.1787	0.1219	0.2377	0.2347	0.2271
2014	0.1855	0.1281	0.2442	0.2468	0.1954
2015	0.1884	0.1344	0.2525	0.2458	0.1788
2016	0.1978	0.1225	0.2426	0.2427	0.1945
2017	0.1868	0.1223	0.2455	0.2484	0.1970

Appendix Table 3 Seven States Ergodic Distribution without Population Weight (Accumulation from 1972)

	ll	lo	ml	mi	mh	hi	hh
1972	0.0385	0.0443	0.0778	0.0639	0.1278	0.2017	0.4459
1973	0.0625	0.0544	0.0703	0.0645	0.1156	0.1946	0.4381
1974	0.0409	0.0366	0.0592	0.0524	0.1015	0.2246	0.4848
1975	0.0369	0.0409	0.0639	0.0562	0.0991	0.2202	0.4828
1976	0.0430	0.0438	0.0688	0.0578	0.1023	0.2279	0.4564
1977	0.0489	0.0459	0.0688	0.0552	0.0925	0.2325	0.4561
1978	0.0548	0.0527	0.0781	0.0580	0.0888	0.2092	0.4584
1979	0.0503	0.0497	0.0668	0.0493	0.0751	0.2059	0.5030
1980	0.0681	0.0623	0.0744	0.0587	0.0810	0.1939	0.4617
1981	0.0576	0.0570	0.0673	0.0537	0.0768	0.1948	0.4929
1982	0.0556	0.0551	0.0643	0.0548	0.0747	0.2000	0.4955
1983	0.0458	0.0453	0.0578	0.0484	0.0714	0.2024	0.5290
1984	0.0622	0.0592	0.0730	0.0609	0.0847	0.2125	0.4474
1985	0.0785	0.0720	0.0911	0.0761	0.0981	0.1998	0.3844
1986	0.0887	0.0742	0.0967	0.0829	0.1120	0.1713	0.3743
1987	0.0913	0.0690	0.0856	0.0769	0.1065	0.1727	0.3982
1988	0.1018	0.0780	0.0876	0.0770	0.1009	0.1619	0.3928
1989	0.1153	0.0829	0.0846	0.0750	0.0991	0.1584	0.3848
1990	0.1326	0.0915	0.0972	0.0819	0.1026	0.1531	0.3412
1991	0.1541	0.0995	0.1014	0.0884	0.1078	0.1438	0.3052
1992	0.1680	0.1103	0.1105	0.0907	0.1067	0.1264	0.2873
1993	0.1791	0.1154	0.1211	0.0955	0.1058	0.1218	0.2614
1994	0.1956	0.1210	0.1232	0.0966	0.0992	0.1169	0.2474
1995	0.2011	0.1213	0.1217	0.0965	0.0997	0.1131	0.2466
1996	0.1933	0.1228	0.1213	0.0942	0.0968	0.1126	0.2590
1997	0.1933	0.1187	0.1235	0.0955	0.0944	0.1125	0.2622
1998	0.2051	0.1282	0.1301	0.0996	0.0945	0.1042	0.2382
1999	0.2072	0.1228	0.1252	0.0995	0.0936	0.1047	0.2471
2000	0.2055	0.1159	0.1202	0.0993	0.0938	0.1078	0.2574
2001	0.1983	0.1162	0.1206	0.1022	0.0954	0.1061	0.2611
2002	0.1815	0.1155	0.1179	0.1009	0.0973	0.1108	0.2761
2003	0.2031	0.1248	0.1226	0.1012	0.0967	0.1043	0.2472
2004	0.1849	0.1209	0.1243	0.1057	0.1048	0.1083	0.2511
2005	0.1732	0.1156	0.1235	0.1081	0.1088	0.1127	0.2581
2006	0.1647	0.1094	0.1165	0.1047	0.1106	0.1157	0.2784
2007	0.1722	0.1085	0.1174	0.1044	0.1096	0.1141	0.2738
2008	0.1705	0.1051	0.1122	0.1056	0.1125	0.1182	0.2760
2009	0.1693	0.1044	0.1129	0.1114	0.1201	0.1162	0.2658
2010	0.1732	0.1064	0.1186	0.1169	0.1234	0.1150	0.2465
2011	0.1655	0.1043	0.1163	0.1182	0.1277	0.1210	0.2470
2012	0.1707	0.1051	0.1174	0.1205	0.1302	0.1191	0.2370
2013	0.1730	0.1078	0.1191	0.1219	0.1353	0.1214	0.2213
2014	0.1836	0.1090	0.1238	0.1255	0.1333	0.1148	0.2100
2015	0.1815	0.1088	0.1259	0.1284	0.1350	0.1166	0.2037
2016	0.1777	0.1096	0.1274	0.1306	0.1375	0.1189	0.1984
2017	0.1820	0.1118	0.1307	0.1334	0.1375	0.1175	0.1870

Appendix Table 4 Seven States Ergodic Distribution without Population Weight (20 Years Moving Average)

	ll	lo	ml	mi	mh	hi	hh
1972	0.0385	0.0443	0.0778	0.0639	0.1278	0.2017	0.4459
1973	0.0543	0.0493	0.0630	0.0642	0.1058	0.1767	0.4867
1974	0.0481	0.0444	0.0707	0.0648	0.1112	0.2122	0.4487
1975	0.0411	0.0495	0.0752	0.0717	0.1119	0.1829	0.4677
1976	0.0555	0.0578	0.0875	0.0753	0.1145	0.1837	0.4257
1977	0.0725	0.0654	0.0894	0.0692	0.0995	0.1836	0.4203
1978	0.0769	0.0709	0.1009	0.0706	0.0948	0.1662	0.4197
1979	0.0700	0.0661	0.0812	0.0534	0.0697	0.1746	0.4850
1980	0.1019	0.0918	0.0989	0.0706	0.0812	0.1554	0.4002
1981	0.0829	0.0830	0.0865	0.0645	0.0827	0.1697	0.4308
1982	0.0768	0.0696	0.0693	0.0497	0.0585	0.1930	0.4832
1983	0.0534	0.0514	0.0578	0.0450	0.0582	0.1997	0.5346
1984	0.0725	0.0693	0.0782	0.0607	0.0738	0.2081	0.4374
1985	0.1089	0.0936	0.0973	0.0814	0.0872	0.1815	0.3501
1986	0.1187	0.0894	0.0997	0.0878	0.1042	0.1566	0.3436
1987	0.1351	0.0837	0.0885	0.0844	0.0971	0.1526	0.3587
1988	0.1427	0.0898	0.0870	0.0812	0.0887	0.1393	0.3712
1989	0.1617	0.0955	0.0825	0.0792	0.0871	0.1368	0.3573
1990	0.2234	0.1106	0.0936	0.0843	0.0829	0.1141	0.2912
1991	0.2500	0.1148	0.0967	0.0924	0.0851	0.1065	0.2544
1992	0.2765	0.1337	0.1082	0.0912	0.0805	0.0879	0.2220
1993	0.2542	0.1395	0.1339	0.1012	0.0847	0.0871	0.1994
1994	0.3306	0.1613	0.1331	0.1010	0.0693	0.0624	0.1423
1995	0.4201	0.1426	0.1122	0.0864	0.0617	0.0531	0.1238
1996	0.3711	0.1532	0.1182	0.0906	0.0623	0.0562	0.1484
1997	0.3615	0.1466	0.1240	0.0978	0.0648	0.0554	0.1499
1998	0.3900	0.1555	0.1242	0.1018	0.0650	0.0473	0.1162
1999	0.4418	0.1407	0.1154	0.1020	0.0622	0.0404	0.0975
2000	0.4004	0.1319	0.1190	0.1072	0.0695	0.0495	0.1225
2001	0.4182	0.1319	0.1196	0.1117	0.0677	0.0429	0.1080
2002	0.3546	0.1448	0.1298	0.1152	0.0788	0.0492	0.1276
2003	0.4038	0.1593	0.1269	0.1116	0.0712	0.0387	0.0886
2004	0.3222	0.1516	0.1344	0.1262	0.0904	0.0486	0.1265
2005	0.2753	0.1394	0.1291	0.1278	0.0990	0.0605	0.1689
2006	0.2501	0.1395	0.1226	0.1245	0.1001	0.0725	0.1905
2007	0.2600	0.1468	0.1385	0.1289	0.0991	0.0653	0.1614
2008	0.2439	0.1248	0.1276	0.1384	0.1168	0.0774	0.1710
2009	0.2071	0.1127	0.1358	0.1620	0.1375	0.0793	0.1657
2010	0.1949	0.1081	0.1326	0.1699	0.1458	0.0839	0.1648
2011	0.1533	0.0982	0.1313	0.1727	0.1601	0.0994	0.1851
2012	0.1421	0.0833	0.1190	0.1852	0.1789	0.1177	0.1737
2013	0.1317	0.0788	0.1022	0.1763	0.2000	0.1328	0.1780
2014	0.1316	0.0718	0.1057	0.1840	0.2128	0.1255	0.1686
2015	0.1207	0.0692	0.1144	0.1934	0.2111	0.1345	0.1567
2016	0.1111	0.0626	0.1108	0.2081	0.2311	0.1442	0.1322
2017	0.1157	0.0670	0.1080	0.2137	0.2394	0.1415	0.1147

Appendix Table 5 Five States Ergodic Distribution with Population Weight (Accumulation from 1972)

	lo	ml	mi	mh	hi
1972	0.0079	0.0113	0.0062	0.0276	0.9471
1973	0.0073	0.0106	0.0066	0.0269	0.9486
1974	0.0060	0.0087	0.0055	0.0265	0.9534
1975	0.0062	0.0090	0.0058	0.0262	0.9529
1976	0.0089	0.0104	0.0058	0.0260	0.9488
1977	0.0100	0.0109	0.0057	0.0258	0.9476
1978	0.0112	0.0143	0.0074	0.0255	0.9416
1979	0.0164	0.0211	0.0103	0.0255	0.9266
1980	0.0230	0.0296	0.0116	0.0247	0.9111
1981	0.0194	0.0250	0.0113	0.0247	0.9197
1982	0.0162	0.0209	0.0118	0.0246	0.9265
1983	0.0394	0.0513	0.0294	0.0623	0.8177
1984	0.0572	0.0742	0.0302	0.0591	0.7794
1985	0.0559	0.0732	0.0304	0.0593	0.7811
1986	0.0665	0.0851	0.0289	0.0568	0.7627
1987	0.0650	0.0827	0.0273	0.0537	0.7714
1988	0.0656	0.0830	0.0275	0.0533	0.7706
1989	0.0695	0.0881	0.0277	0.0524	0.7622
1990	0.0734	0.0928	0.0290	0.0515	0.7534
1991	0.0828	0.1042	0.0322	0.0547	0.7261
1992	0.0872	0.1097	0.0329	0.0553	0.7150
1993	0.0830	0.1040	0.0360	0.0526	0.7245
1994	0.0871	0.1084	0.0370	0.0512	0.7164
1995	0.0661	0.0817	0.0347	0.0543	0.7632
1996	0.0618	0.0760	0.0340	0.0552	0.7730
1997	0.0667	0.0811	0.0363	0.0546	0.7612
1998	0.1080	0.1310	0.0447	0.0573	0.6590
1999	0.1257	0.1517	0.0492	0.0537	0.6197
2000	0.1268	0.1520	0.0484	0.0520	0.6207
2001	0.1275	0.1521	0.0488	0.0516	0.6199
2002	0.0711	0.0835	0.0551	0.0604	0.7299
2003	0.0658	0.0762	0.0535	0.0611	0.7434
2004	0.0651	0.0746	0.0550	0.0613	0.7440
2005	0.0578	0.0751	0.0572	0.0616	0.7484
2006	0.0546	0.0717	0.0571	0.0626	0.7540
2007	0.0556	0.0737	0.0597	0.0622	0.7487
2008	0.0550	0.0737	0.0613	0.0623	0.7478
2009	0.0502	0.0678	0.0640	0.0639	0.7541
2010	0.0534	0.0731	0.0688	0.0730	0.7316
2011	0.0546	0.0754	0.0738	0.0773	0.7189
2012	0.0528	0.0735	0.0736	0.0783	0.7217
2013	0.0519	0.0726	0.0744	0.0792	0.7219
2014	0.0550	0.0774	0.0794	0.0786	0.7096
2015	0.0589	0.0836	0.0874	0.0778	0.6924
2016	0.0567	0.0808	0.0888	0.0776	0.6961
2017	0.0572	0.0820	0.0920	0.0791	0.6897

Appendix Table 6 Five States Ergodic Distribution with Population Weight (20 Years Moving Average)

	lo	ml	mi	mh	hi
1972	0.0079	0.0113	0.0062	0.0276	0.9471
1973	0.0069	0.0101	0.0066	0.0268	0.9496
1974	0.0080	0.0092	0.0056	0.0259	0.9513
1975	0.0065	0.0089	0.0058	0.0254	0.9534
1976	0.0098	0.0100	0.0058	0.0249	0.9494
1977	0.0159	0.0150	0.0081	0.0349	0.9262
1978	0.0261	0.0295	0.0155	0.0355	0.8933
1979	0.0387	0.0435	0.0219	0.0353	0.8605
1980	0.1341	0.1205	0.0281	0.0279	0.6894
1981	0.0651	0.0720	0.0220	0.0316	0.8092
1982	0.0041	0.0046	0.0021	0.0028	0.9864
1983	0.0755	0.0859	0.0396	0.0544	0.7446
1984	0.1304	0.1107	0.0379	0.0478	0.6732
1985	0.1246	0.1229	0.0419	0.0466	0.6640
1986	0.1309	0.1301	0.0357	0.0439	0.6594
1987	0.1257	0.1230	0.0348	0.0399	0.6767
1988	0.0761	0.1276	0.0371	0.0409	0.7182
1989	0.1462	0.2647	0.0654	0.0688	0.4549
1990	0.2373	0.2615	0.0599	0.0583	0.3829
1991	0.3197	0.3490	0.0759	0.0620	0.1935
1992	0.3046	0.3572	0.0756	0.0659	0.1967
1993	0.2969	0.3118	0.0777	0.0585	0.2552
1994	0.3281	0.3390	0.0837	0.0445	0.2047
1995	0.2651	0.2623	0.0948	0.0674	0.3105
1996	0.0811	0.2872	0.1207	0.0938	0.4172
1997	0.0868	0.3017	0.1288	0.0900	0.3927
1998	0.3937	0.2921	0.0863	0.0524	0.1755
1999	0.4172	0.3013	0.0859	0.0461	0.1494
2000	0.4058	0.2847	0.0860	0.0479	0.1756
2001	0.4250	0.2944	0.0831	0.0423	0.1552
2002	0.2995	0.1847	0.1393	0.0805	0.2960
2003	0.2080	0.1085	0.0933	0.0574	0.5329
2004	0.1808	0.0900	0.1042	0.0617	0.5633
2005	0.0496	0.1009	0.1280	0.0702	0.6513
2006	0.0229	0.0885	0.1482	0.0813	0.6591
2007	0.0270	0.1160	0.2106	0.1042	0.5424
2008	0.0215	0.1142	0.2217	0.1051	0.5375
2009	0.0166	0.0970	0.2432	0.1121	0.5312
2010	0.0171	0.1071	0.2740	0.1530	0.4488
2011	0.0112	0.0995	0.2838	0.1585	0.4469
2012	0.0097	0.0898	0.2820	0.1619	0.4567
2013	0.0112	0.1080	0.3473	0.2412	0.2923
2014	0.0117	0.1180	0.4022	0.2533	0.2147
2015	0.0132	0.1510	0.4806	0.1966	0.1586
2016	0.0132	0.1416	0.4854	0.1761	0.1837
2017	0.0094	0.1329	0.4879	0.1862	0.1836

Appendix Table 7 Seven States Ergodic Distribution with Population Weight (Accumulation from 1972)

	ll	lo	ml	mi	mh	hi	hh
1972	0.0003	0.0028	0.0011	0.0019	0.0084	0.0685	0.9171
1973	0.0005	0.0040	0.0011	0.0020	0.0081	0.0672	0.9172
1974	0.0014	0.0081	0.0025	0.0045	0.0219	0.1911	0.7705
1975	0.0011	0.0080	0.0019	0.0037	0.0173	0.1523	0.8155
1976	0.0016	0.0088	0.0021	0.0039	0.0179	0.1565	0.8091
1977	0.0018	0.0089	0.0020	0.0036	0.0168	0.1468	0.8201
1978	0.0039	0.0170	0.0038	0.0059	0.0208	0.1410	0.8077
1979	0.0043	0.0173	0.0033	0.0051	0.0131	0.1032	0.8536
1980	0.0074	0.0275	0.0049	0.0070	0.0152	0.0977	0.8403
1981	0.0064	0.0226	0.0043	0.0064	0.0145	0.0955	0.8503
1982	0.0064	0.0214	0.0042	0.0066	0.0140	0.0924	0.8550
1983	0.0042	0.0133	0.0040	0.0062	0.0134	0.0906	0.8683
1984	0.0090	0.0270	0.0083	0.0126	0.0251	0.1687	0.7493
1985	0.0088	0.0251	0.0080	0.0119	0.0235	0.1585	0.7641
1986	0.0103	0.0245	0.0180	0.0229	0.0450	0.1454	0.7339
1987	0.0105	0.0234	0.0193	0.0220	0.0436	0.1421	0.7390
1988	0.0169	0.0372	0.0200	0.0217	0.0426	0.1360	0.7255
1989	0.0424	0.0925	0.0202	0.0205	0.0391	0.1229	0.6625
1990	0.0483	0.1005	0.0226	0.0225	0.0405	0.1195	0.6461
1991	0.0535	0.1069	0.0241	0.0238	0.0407	0.1189	0.6321
1992	0.0599	0.1166	0.0263	0.0244	0.0410	0.1133	0.6186
1993	0.0655	0.1214	0.0435	0.0397	0.0576	0.1041	0.5682
1994	0.0611	0.1079	0.0417	0.0391	0.0542	0.1060	0.5899
1995	0.0660	0.1120	0.0463	0.0402	0.0629	0.1012	0.5713
1996	0.0672	0.1099	0.0485	0.0383	0.0632	0.0996	0.5732
1997	0.0740	0.1165	0.0551	0.0396	0.0617	0.0954	0.5577
1998	0.0822	0.1278	0.0641	0.0440	0.0598	0.0897	0.5324
1999	0.0934	0.1379	0.0730	0.0489	0.0569	0.0841	0.5059
2000	0.0936	0.1349	0.0752	0.0493	0.0565	0.0831	0.5074
2001	0.0925	0.1288	0.0755	0.0490	0.0552	0.0809	0.5181
2002	0.0698	0.1244	0.0762	0.0485	0.0567	0.0831	0.5412
2003	0.0736	0.1315	0.0756	0.0476	0.0602	0.0874	0.5241
2004	0.0682	0.1254	0.0783	0.0482	0.0620	0.0875	0.5305
2005	0.0653	0.1211	0.0781	0.0491	0.0634	0.0890	0.5341
2006	0.0600	0.1113	0.0738	0.0468	0.0635	0.0872	0.5575
2007	0.0594	0.1098	0.0770	0.0478	0.0636	0.0858	0.5567
2008	0.0574	0.1055	0.0719	0.0470	0.0629	0.0933	0.5621
2009	0.0386	0.0687	0.0464	0.0614	0.0823	0.0998	0.6029
2010	0.0376	0.0670	0.0453	0.0631	0.0885	0.1164	0.5822
2011	0.0402	0.0715	0.0478	0.0756	0.1032	0.1353	0.5264
2012	0.0389	0.0693	0.0458	0.0763	0.1036	0.1379	0.5283
2013	0.0396	0.0708	0.0465	0.0810	0.1083	0.1391	0.5147
2014	0.0349	0.0605	0.0499	0.0908	0.1111	0.1404	0.5123
2015	0.0408	0.0706	0.0574	0.1049	0.1137	0.1338	0.4789
2016	0.0412	0.0710	0.0588	0.1102	0.1157	0.1371	0.4660
2017	0.0404	0.0711	0.0602	0.1144	0.1166	0.1378	0.4595

Appendix Table 8 Seven States Ergodic Distribution with Population Weight (20 Years Moving Average)

	ll	lo	ml	mi	mh	hi	hh
1972	0.0003	0.0028	0.0011	0.0019	0.0084	0.0685	0.9171
1973	0.0004	0.0032	0.0009	0.0016	0.0067	0.0572	0.9300
1974	0.0032	0.0172	0.0054	0.0095	0.0449	0.2810	0.6387
1975	0.0019	0.0131	0.0032	0.0062	0.0282	0.1755	0.7717
1976	0.0030	0.0147	0.0036	0.0064	0.0295	0.1817	0.7611
1977	0.0035	0.0146	0.0033	0.0056	0.0266	0.1641	0.7822
1978	0.0109	0.0391	0.0093	0.0130	0.0329	0.1573	0.7374
1979	0.0107	0.0336	0.0067	0.0093	0.0166	0.1078	0.8153
1980	0.0319	0.0885	0.0167	0.0210	0.0228	0.0956	0.7235
1981	0.0138	0.0545	0.0102	0.0145	0.0222	0.0997	0.7851
1982	0.0240	0.0501	0.0081	0.0112	0.0154	0.0952	0.7961
1983	0.0137	0.0260	0.0074	0.0116	0.0165	0.0941	0.8307
1984	0.0300	0.0515	0.0158	0.0245	0.0315	0.1784	0.6684
1985	0.0319	0.0498	0.0159	0.0250	0.0279	0.1591	0.6904
1986	0.0575	0.0702	0.0578	0.0765	0.0919	0.1260	0.5203
1987	0.0711	0.0664	0.0668	0.0767	0.0876	0.1165	0.5149
1988	0.1199	0.1098	0.0850	0.0676	0.0759	0.0944	0.4473
1989	0.1962	0.1812	0.0569	0.0572	0.0636	0.0742	0.3706
1990	0.3150	0.2268	0.0727	0.0473	0.0480	0.0454	0.2448
1991	0.3347	0.2194	0.0735	0.0467	0.0395	0.0440	0.2421
1992	0.3598	0.2209	0.0767	0.0430	0.0376	0.0381	0.2239
1993	0.3667	0.2173	0.1338	0.0611	0.0451	0.0254	0.1506
1994	0.2434	0.2121	0.1548	0.0809	0.0427	0.0280	0.2381
1995	0.7481	0.0729	0.0629	0.0247	0.0172	0.0086	0.0657
1996	0.4269	0.1453	0.1498	0.0529	0.0416	0.0196	0.1638
1997	0.4384	0.1447	0.1740	0.0523	0.0385	0.0167	0.1354
1998	0.3361	0.1552	0.2142	0.0682	0.0456	0.0191	0.1616
1999	0.4485	0.1450	0.2322	0.0702	0.0417	0.0112	0.0512
2000	0.3855	0.1404	0.2599	0.0829	0.0516	0.0143	0.0654
2001	0.4574	0.1171	0.2492	0.0721	0.0414	0.0100	0.0528
2002	0.1220	0.1729	0.4272	0.1047	0.0689	0.0168	0.0876
2003	0.1270	0.1823	0.4139	0.1043	0.0806	0.0167	0.0753
2004	0.0958	0.1509	0.3844	0.0930	0.0763	0.0149	0.1847
2005	0.0962	0.1519	0.4088	0.1102	0.0933	0.0173	0.1223
2006	0.0690	0.1694	0.2973	0.1081	0.1001	0.0248	0.2315
2007	0.0637	0.1815	0.3137	0.1140	0.1020	0.0217	0.2034
2008	0.0574	0.1603	0.2823	0.1197	0.1088	0.0380	0.2336
2009	0.0148	0.0242	0.1207	0.2325	0.2135	0.0553	0.3389
2010	0.0118	0.0205	0.1029	0.2351	0.2482	0.0898	0.2917
2011	0.0034	0.0155	0.0971	0.2990	0.3048	0.1003	0.1799
2012	0.0022	0.0103	0.0703	0.3161	0.3187	0.1181	0.1642
2013	0.0045	0.0218	0.0471	0.2534	0.2979	0.1671	0.2082
2014	0.0018	0.0090	0.0499	0.3172	0.3217	0.1437	0.1567
2015	0.0024	0.0128	0.0559	0.3978	0.2536	0.1412	0.1363
2016	0.0013	0.0075	0.0317	0.4533	0.2429	0.1476	0.1157
2017	0.0006	0.0038	0.0157	0.4625	0.2476	0.1584	0.1113